# Effects of interfacial bonding on sliding phenomena during compressive loading of an embedded fibre

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The effects of interfacial bonding on the sliding phenomena at the fibre/matrix interface are considered for fibre-reinforced ceramic composites when an axial compressive stress is applied at the exposed end of an embedded fibre. Sliding occurs at the interface when the interfacial shear strength is exceeded. The interfacial shear stress, the stress transfer from the fibre to the matrix, the length of the sliding zone, and the fibre displacement are analysed in the present study. The results show that at a fixed load, the sliding length and the fibre displacement decrease with increase in the interfacial shear strength. Effects of interfacial bonding on the applied stress–fibre displacement relationship during compressive loading and subsequent unloading are also revealed.

#### 1. Introduction

The properties at the fibre/matrix interface influence the mechanical behaviour of fibre-reinforced ceramic composites. One of the most significant toughening mechanisms in such composites is the bridging of the crack surfaces by intact fibres when the composite is subjected to tension. While the bridging stresses in fibres contribute to the toughening effect, relative displacement in the loading direction between fibres and the matrix at the crack surface is required to accommodate the crack opening displacement. Consequently, optimal conditions for toughening of these composites require debonding at fibre/matrix interfaces by shear during fibre pullout, and allow frictional sliding between fibres and the matrix [1-3]. These requirements have prompted studies of interfacial properties of composites: the interfacial shear strength associated with debonding and, after debonding, the interfacial frictional stress [4-14].

The effects of the coefficient of friction, the residual clamping stress at the interface, and the elastic constants of the constituent materials on the interfacial sliding phenomena of composites have been analysed theoretically for an embedded fibre subjected to an applied stress at its exposed end along its axial direction [12-14]. However, those results are based on the condition of an unbonded and frictional interface, which maximizes the toughening effect. In general, chemical or mechanical bonding is often unavoidable at the interface during processing of the composite [14]. As a complement of previous studies [12–14], the effects of interfacial bonding on the sliding phenomena are considered in the present work. First, the solution of the fibre displacement is given when the interface is bonded, and the condition for interfacial debonding is addressed. Then, when the interface is

partially debonded, the interfacial shear stress, the stress transfer from the fibre to the matrix, the length of the sliding zone, and the fibre displacement are analysed. Finally, the effects of interfacial bonding on the applied stress-fibre displacement relationship during compressive loading and subsequent unloading are shown.

#### 2. Analyses

The geometry of the system considered in the present study is a composite cylinder model [15], which is also used in previous studies [12–14]. A semi-infinitely long fibre with a radius, a, is located at the centre of a coaxial cylindrical shell of the matrix with an outer radius, b. A bonded fibre/matrix interface is considered in the present analysis. As shown in Fig. 1, z is the direction parallel to the fibre axis and r is the radial distance from the fibre axis. An axial compressive stress,  $\sigma_0$  (negative for compression), is applied at the exposed end of the embedded fibre (i.e. at z = l in Fig. 1). For convenience, when comparing compressive stresses, the negative sign is omitted and only the magnitude is implied in the text. When the applied axial compressive stress is lower than that of a critical compressive stress,  $\sigma_d$ , the bonded interface remains intact (Fig. 1a). However, when the applied axial stress exceeds this critical stress, debonding and sliding are initiated at the surface and extend to a depth l beneath the surface, such that the end of the sliding zone located at z = 0 (Fig. 1b).

#### 2.1. Bonded interface

The problem of the elastic stress transfer from an embedded fibre to the surrounding matrix for the case of a bonded interface has been analysed [3]. When the fibre is subjected to an axial compressive stress ( $\sigma_0$ ) at



its exposed end, the stress is transferred from the fibre to the matrix through the interfacial shear stress. Relative displacement between the fibre and the matrix is prohibited at the interface when the interface is bonded. However, a relative displacement exists between the fibre and the bulk of the matrix far from the interface. The maximum relative displacement,  $u_b$ , occurs at the surface (see Fig. 1a) and is related to the applied axial stress,  $\sigma_0$ , by [3]

$$u_{\rm b} = \frac{a(b^2 - a^2)E_{\rm m}\sigma_0}{a^2E_{\rm f} + (b^2 - a^2)E_{\rm m}} \times \left\{ \frac{(1 + v_{\rm m})[a^2 - b^2 + b^2\ln(b^2/a^2)]}{2E_{\rm f}[a^2E_{\rm f} + (b^2 - a^2)E_{\rm m}]} \right\}^{1/2}$$
(1)

where E and v are Young's modulus and Poisson's ratio, respectively, and the subscripts f and m denote the fibre and the matrix, respectively. The interfacial shear stress has a maximum value at the surface (z = l). Debonding is expected to initiate at the surface when the axially applied stress,  $\sigma_0$ , reaches a critical value,  $\sigma_d$ , for which the corresponding maximum interfacial shear stress reaches the interfacial shear strength,  $\tau_s$ . The interfacial shear strength,  $\tau_s$ , can then be related to  $\sigma_d$  by [3]

$$\tau_{\rm s} = -(b^2 - a^2) E_{\rm m} \sigma_{\rm d} \{ 2E_{\rm f} (1 + v_{\rm m}) \\ \times [a^2 E_{\rm f} + (b^2 - a^2) E_{\rm m}] \\ \times [a^2 - b^2 + b^2 \ln (b^2/a^2)] \}^{-1/2}$$
(2)

#### 2.2. Interfacial debonding and sliding

When the interfacial shear strength is exceeded, debonding and frictional sliding occur at the interface. The solutions of fibre push-down subject to a residual clamping stress,  $\sigma_c$ , at the interface and an axial pushdown stress,  $\sigma_0$ , have been derived for an unbonded interface [12]. In the presence of interfacial bonding, the boundary condition at the end of sliding needs to be modified as compared to the unbonded case. Without repeating the calculational procedures [12], the present paper gives only the differences required in deriving the solutions for the bonded case.

The equation governing the axial stress distribution

Figure 1 Schematic illustration of the composite cylinder model for (a) a bonded interface when the applied compressive stress is low ( $|\sigma_0| \leq |\sigma_d|$ ), and (b) debonding and sliding when  $|\sigma_0| > |\sigma_d|$ , also showing the fibre displacement.

in the fibre,  $\sigma_{\rm f}$ , along the sliding length has been derived previously [12], such that

$$\sigma_{\rm f} = \frac{A_3}{A_2} + X_1 \exp(m_1 z) + X_2 \exp(m_2 z) \quad (3)$$

where the coefficients,  $A_2$ ,  $A_3$ ,  $m_1$  and  $m_2$  are given by

$$A_{2} = \frac{2\left[\left(1 - \frac{b^{2}}{a^{2}}\right)\frac{E_{m}v_{f}}{E_{f}v_{m}} - 1\right]}{(1 + v_{m})[a^{2} - b^{2} + b^{2}\ln(b^{2}/a^{2})]}$$
(4)

$$A_{3} = 2 \left\{ \sigma_{0} + \left(1 - \frac{b^{2}}{a^{2}}\right) \left[ 1 + \frac{b^{2} + a^{2}}{(b^{2} - a^{2})v_{m}} + \frac{E_{m}(1 - v_{f})}{E_{f}v_{m}} \right] \sigma_{c} \right\} \right|$$

$$\times (1 + v_{m})[a^{2} - b^{2} + b^{2} \ln (b^{2}/a^{2})] \quad (5)$$

$$\int (A^2 + (A^2 + A^2)^{1/2})/2 = (6)$$

$$m_{1} = \begin{bmatrix} x_{1} + (x_{1} + x_{2}) \end{bmatrix} / 2$$
 (0)

$$m_2 = \left[-A_1 - (A_1^2 - 4A_2)^{1/2}\right]/2$$
(7)

and  $A_1$  is given by

$$A_{1} = \frac{a\left(1 - \frac{b^{2}}{a^{2}}\right)\left[1 + \frac{b^{2} + a^{2}}{(b^{2} - a^{2})v_{m}} + \frac{E_{m}(1 - v_{f})}{E_{f}v_{m}}\right]}{\mu(1 + v_{m})[a^{2} - b^{2} + b^{2}\ln(b^{2}/a^{2})]}$$
(8)

where  $\mu$  is the coefficient of friction at the sliding interface. The coefficients,  $X_1$  and  $X_2$ , in Equation 3 can be determined from the following boundary conditions

$$\sigma_{\rm f} = \sigma_0 \quad (\text{at } z = l) \tag{9}$$

$$\sigma_{\rm f} = \sigma_{\rm d} \quad ({\rm at} \ z = 0) \tag{10}$$

Equations 9 and 10 dictate that axial stresses in the fibre are in equilibrium with the applied stress and the bonding strength at the loaded surface and the end of sliding, respectively. Solution of  $\sigma_f$  (Equation 3) subject to the boundary conditions given by Equations 9 and 10, yields

$$X_{1} = \frac{\sigma_{0} - \sigma_{d} \exp(m_{2}l) - (A_{3}/A_{2})[1 - \exp(m_{2}l)]}{\exp(m_{1}l) - \exp(m_{2}l)}$$
(11)

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$$X_2 = \sigma_{\rm d} - \frac{A_3}{A_2} - X_1 \tag{12}$$

The corresponding interfacial shear stress  $(\tau_i)$ , the interfacial radial stress  $(\sigma_r)$ , the axial stress in the matrix at the interface  $(\sigma_a)$ , and the axial displacement in the fibre  $(w_f)$  due to sliding are [12]

$$\tau_{i} = -\frac{a}{2} \left\{ -\frac{m_{2}A_{3} \exp(m_{2}z)}{A_{2}} + X_{1}[m_{1} \exp(m_{1}z) - m_{2} \exp(m_{2}z)] + \sigma_{d}m_{2} \exp(m_{2}z) \right\}$$
(13)

$$\sigma_{\rm r} = \frac{\tau_{\rm i}}{\mu} - \sigma_{\rm c} \tag{14}$$

$$\sigma_{a} = -\left[1 + \frac{b^{2} + a^{2}}{(b^{2} - a^{2})v_{m}} + \frac{E_{m}(1 - v_{f})}{E_{f}v_{m}}\right]\sigma_{r}$$
$$+ \frac{E_{m}v_{f}\sigma_{f}}{E_{f}v_{m}}$$
(15)

$$w_{\rm f} = \frac{1}{E_{\rm f}} \left\{ \frac{A_3 z}{A_2} + \frac{A_3 [1 - \exp(m_2 z)]}{m_2 A_2} + X_1 \left[ \frac{\exp(m_1 z) - 1}{m_1} - \frac{\exp(m_2 z) - 1}{m_2} \right] + \frac{\sigma_{\rm d} [\exp(m_2 z) - 1]}{m_2} \right\}$$
(16)

The axial displacement in the fibre due to sliding,  $w_{\rm f}$ , has a maximum value,  $u_{\rm s}$ , at the surface (see Fig. 1b). Solutions of the interfacial properties given by Equations 3, 13 to 16 are contingent upon the solution of the length of the sliding zone, l, which can be determined from the continuity condition at the end of the sliding zone.

At the end of sliding, the relative displacement between the fibre and the matrix at the interface is prohibited. This condition requires continuity of the axial strains of the fibre and the matrix at the end of sliding, such that [12]

$$\frac{1}{E_{\rm f}}[\sigma_{\rm d} - 2\nu_{\rm f}\sigma_{\rm r}] = \frac{1}{E_{\rm m}}\left[\sigma_{\rm a} + \frac{2a^2\nu_{\rm m}\sigma_{\rm r}}{b^2 - a^2}\right] \quad (\text{at } z = 0)$$
(17)

Substitutions of  $\sigma_r$  and  $\sigma_a$  given by Equations 14 and 15 in Equation 17 yields the relationship between the length of the sliding zone, *l*, and the axial stress required for debonding,  $\sigma_d$ , such that Hence, with a given value of the interfacial shear strength,  $\tau_s$ , the corresponding axial stress in the fibre required to initiate debonding,  $\sigma_d$ , can be obtained from Equation 2. With  $\sigma_d$  obtained, the length of the sliding zone, *l*, can be derived from Equation 18. With the solution of *l*, the descriptions of the interfacial properties (Equations 3, 13 to 16) are complete.

#### 3. Results

Specific results are computed using the material properties of SiC fibre-reinforced Al<sub>2</sub>O<sub>3</sub> with  $E_{\rm f} = 500$  GPa,  $E_{\rm m} = 400$  GPa,  $v_{\rm f} = 0.3$ , and  $v_{\rm m} = 0.25$ . The system, was also chosen previously [12–14]. Also, b/a = 10 (the results are not sensitive to b/a when  $b/a \ge 10$ ),  $\sigma_c/\sigma_0 = 0.1$  and  $\mu = 0.1$  are used in the present calculation to elucidate the essential trends. Effects of interfacial bonding on sliding phenomena are shown by assuming arbitrarily the ratio of the interfacial shear strength to the applied axial stress,  $\tau_s/\sigma_0$ .

## 3.1. Conditions for bonding and debonding

The ratio,  $\tau_s/\sigma_0$ , is an important parameter in determining the bonding and the debonding conditions. With a fixed interfacial shear strength,  $\tau_s$ , for a composite, the corresponding axial stress required to initiate debonding,  $\sigma_d$ , can be obtained from Equation 2. When the applied stress is higher than the value of  $\sigma_d$ , interfacial debonding and sliding occur. Otherwise, the interface remains intact. Modifications of the material properties to control the  $\tau_s$ - $\sigma_d$  relationship, and hence the fracture mechanism of the composite, have been addressed in a separate paper [3].

The normalized fibre displacement component due to the bonded interface,  $u_b/a\sigma_0$  (where  $\sigma_0$  is expressed in GPa), and the normalized axial stress in the fibre at the end of sliding,  $\sigma_d/\sigma_0$ , as functions of the normalized interfacial shear strength,  $-\tau_s/\sigma_0$ , are shown in Fig. 2. For the chosen materials properties (i.e. SiC– Al<sub>2</sub>O<sub>3</sub>), as the ratio of  $-\tau_s/\sigma_0$  increases and reaches 0.29 (see Equation 2), the corresponding stress required for debonding,  $\sigma_d$ , increases and reaches the applied axial stress,  $\sigma_0$ . Hence, when  $-\tau_s/\sigma_0 \ge 0.29$ , the interface remains intact and the fibre displacement,  $u_b$ , is linearly proportional to the applied stress. However, when  $-\tau_s/\sigma_0 < 0.29$ , debonding and sliding occur such that both  $\sigma_d$  and  $u_b$  are linearly proportional to  $\tau_s$  (see Fig. 2).

# 3.2. Effects of interfacial shear strength on sliding phenomena

The length of the sliding zone as a function of the

$$\begin{bmatrix} 2\left(\frac{a^{2}v_{m}}{b^{2}-a^{2}}+\frac{E_{m}v_{f}}{E_{f}}\right)-1-\frac{b^{2}+a^{2}}{(b^{2}-a^{2})v_{m}}-\frac{E_{m}(1-v_{f})}{E_{f}v_{m}}\end{bmatrix} \\ \times \left\{\frac{a}{2\mu}\left[\frac{m_{2}A_{3}}{A_{2}}-\frac{(m_{1}-m_{2})[\sigma_{0}-(A_{3}/A_{2})[1-\exp(m_{2}l)]]}{\exp(m_{1}l)-\exp(m_{2}l)}\right]-\sigma_{c}\right\} \\ = \left\{\frac{E_{m}}{E_{f}}\left(1-\frac{v_{f}}{v_{m}}\right)+\frac{a}{2\mu}\left[2\left(\frac{a^{2}v_{m}}{b^{2}-a^{2}}+\frac{E_{m}v_{f}}{E_{f}}\right)-1-\frac{b^{2}+a^{2}}{(b^{2}-a^{2})v_{m}}-\frac{E_{m}(1-v_{f})}{E_{f}v_{m}}\right] \\ \times \left[m_{2}-\frac{(m_{1}-m_{2})\exp(m_{2}l)}{\exp(m_{1}l)-\exp(m_{2}l)}\right]\right\}\sigma_{d}$$
(18)

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Figure 2 Normalized fibre displacement due to a bonded interface,  $u_{\rm b}/a\sigma_0$ , and normalized axial stress in the fibre at the end of sliding,  $\sigma_{\rm d}/\sigma_0$ , as functions of normalized interfacial shear strength,  $-\tau_s/\sigma_0$ , for  $\sigma_{\rm c}/\sigma_0 = 0.1$  and  $\mu = 0.1$ .

interfacial shear strength is shown in Fig. 3. At a fixed applied axial stress, the sliding length decreases with an increase in the interfacial shear strength. Effects of the interfacial shear strength on the axial stress distribution in the fibre and the interfacial shear stress along the sliding length, obtained from Equations 3 and 13, are shown in Figs 4a and b, respectively. The stress distributions within the sliding length are not sensitive to  $\tau_s$ . However, the sliding length decreases with an increase in  $\tau_s$ . The axial stress in the fibre is the applied axial stress at the loaded surface and decreases with the distance beneath the surface because of the stress transfer. Also, the interfacial shear stress is not constant along the sliding length because of Poisson's effect, which has been discussed in previous studies [12-14].



*Figure 3* Normalized length of the sliding zone, l/a, as a function of normalized interfacial shear strength,  $-\tau_s/\sigma_0$ , for  $\sigma_c/\sigma_0 = 0.1$  and  $\mu = 0.1$ .

For a bonded interface, the relationship between the fibre displacement and the applied axial stress is given by Equation 1 (also see Fig. 1a). When debonding and sliding occur, the total fibre displacement, u, consists of two components (see Fig. 1b): (1) the displacement,  $u_{\rm b}$ , due to the bonded interface which is induced by the stress at the end of sliding (i.e.  $\sigma_d$  at z = 0), and (2) the displacement,  $u_s$ , due to relative sliding between the fibre and the matrix. The normalized fibre displacements are shown in Fig. 5 as functions of the normalized interfacial shear strength. The fibre displacement due to the bonded interface,  $u_{\rm b}$ , was discussed in Section 2.1. At a fixed load, the fibre displacement due to sliding,  $u_s$ , decreases with an increase in  $\tau_s$ , and is zero when sliding is prohibited (i.e. when  $\tau_s/\sigma_0 > 0.29$  in Fig. 5). The resultant fibre displacement,  $u (= u_b + u_s)$ , decreases with an increase in  $\tau_s$  (see Fig. 5).

## 3.3. Effects of the interfacial shear strength on axial loading and unloading

The stress-fibre displacement relationship during axial compressive loading and subsequent unloading has been derived for the case of an unbonded interface [14]. Those results show that in the absence of Poisson's effect (i.e.  $v_f = 0$ ), the loading curve is parabolic, and after complete unloading, the residual fibre displacement equals half of the fibre displacement at the peak load [5, 14]. When Poisson's effect increases (i.e. when  $v_{\rm f}$  increases or the ratio of  $\sigma_{\rm c}/\sigma^*$ decreases, where  $\sigma^*$  is the peak load), the loading curve shows a transition from parabolic to linear behaviour, and the ratio of the fibre displacement at complete unloading to that at peak loading (i.e.  $u_0/u^*$ in Fig. 6a) decreases from 0.5 to lower values [14]. A method for evaluating the coefficient of friction and the residual clamping stress from the loading and the unloading curves has also been demonstrated [14].

Effects of the interfacial shear strength on the load-



Figure 4 (a) Normalized axial stress in the fibre,  $\sigma_f/\sigma_0$ , and (b) normalized interfacial shear stress,  $-\tau_i/\sigma_0$ , along the sliding length, also showing the depth corresponding to the end of sliding for different ratios of  $-\tau_s/\sigma_0$ .

ing and the unloading curves are shown in Fig. 6a for  $\sigma_c/\sigma^* = 0.1$ ,  $\mu = 0.1$  and  $-\tau_s/\sigma^* = 0.05$ . For a bonded interface, when the applied compressive stress is lower than  $\sigma_d$ , the interface remains intact and the  $\sigma_0$ -u relationship is linear. As the compressive stress increases and exceeds  $\sigma_d$ , interfacial debonding and forward sliding of the fibre occur and the solutions are given by Equations 3, 13 to 16. During unloading, reverse sliding of the fibre occurs beginning at the surface and extending along the interface beneath the surface. However, the length of the reverse sliding zone at complete unloading is always less than the



length of the forward sliding zone at peak loading [14]. Hence, even if the interface is bonded initially, reverse sliding occurs within the zone of the debonded interface resulting from the peak loading stress,  $\sigma^*$ , prior to unloading. The solutions derived in the previous study for unloading of an embedded fibre with an unbonded interface [14] remain valid for the case of an initially bonded interface.

Compared to the case of an unbonded interface, the loading curve shows an initial linear relationship with a lower compliance for the bonded case. Also, after complete unloading, the  $u_0/u^*$  ratio has a lower value

*Figure 5* Normalized fibre displacement as a function of normalized interfacial shear strength for  $\sigma_c/\sigma_0 = 0.1$  and  $\mu = 0.1$ .



Figure 6 Effects of the interfacial shear strength on (a) the loading and the unloading curves, and (b) the lengths of the forward and the reverse sliding zones when  $(--) - \tau_s / \sigma^* = 0.05$ ,  $\sigma_c / \sigma^* = 0.1$  and  $\mu = 0.1$ . (--) Unbonded interface.

for the bonded case. Effects of the interfacial shear strength on sliding lengths are shown in Fig. 6b for  $\sigma_c/\sigma^* = 0.1$ ,  $\mu = 0.1$  and  $-\tau_s/\sigma^* = 0.05$ . For the bonded interface, a minimum stress,  $\sigma_d$ , is required to initiate forward sliding and the length of the forward sliding zone is shorter compared to the case of an unbonded interface. However, the length of the reverse sliding zone is the same for both the bonded and the unbonded interfaces (see Fig. 6b) because of the reason stated earlier.

### 3.4. Applications

It is noted that adoption of the composite cylinder model for the present analysis requires low volume fractions of fibres (e.g.  $a^2/b^2 \leq 0.01$ ). In the presence of high volume fractions of fibres, the self-consistent model should be applied where the fibre is surrounded by a material, which has the properties of the composite [16]. The stress-displacement relationship of the fibre for Nicalon fibre-reinforced lithium aluminosilicate glass-ceramic was obtained by using the indentation technique (see Fig. 7) [5]. The properties of the materials are:  $E_{\rm f} = 200 \,\rm{GPa}, E_{\rm m} = 80 \,\rm{GPa}$ ,  $v_{\rm f} = 0.15$  and  $v_{\rm m} = 0.3$  and the volume fraction of fibres is 0.3. Owing to the high volume fraction of fibres, the self consistent model is adopted, and the rule of mixtures is used to derive elastic constants of the composite to replace  $E_{\rm m}$  and  $v_{\rm m}$  in the present analysis (i.e.  $E_{\rm m}$  and  $v_{\rm m}$  become 116 GPa and 0.255, respectively). Comparison between the theoretical prediction and the experimental results shows that good agreement is obtained when  $\sigma_d = -78$  MPa,  $\sigma_{\rm c} = -21.5 \, {\rm MPa}$  and  $\mu = 0.09$  are used in the theoretical prediction (see Fig. 7).

#### 4. Discussion

The effects of interfacial bonding on the interfacial sliding phenomena of fibre-reinforced ceramic composites are shown in the present study. A simple interfacial debonding criterion is adopted by considering that debonding occurs when the maximum interfacial shear stress exceeds the interfacial shear strength,  $\tau_s$  [4]. The residual clamping stress is not considered in deriving the stress required to initiate debonding,  $\sigma_d$ , for which the corresponding maximum interfacial shear stress reaches  $\tau_s$  [3]. Further studies are required to establish pertinent debonding criteria. In the case of a multiaxial interfacial debonding criterion, the effect of the residual clamping stress on the interfacial shear strength should be included.

Experiments of axial compressive loading and subsequent unloading performed on Nicalon fibrereinforced glass show the evidence of the interfacial bonding. At initial loading, displacement of the fibre in the loading direction indicates a higher resistance to movement (i.e. lower compliance). Then, when the loading stress exceeds a critical value, the loading curve shows a higher compliance. The experimental loading/unloading curves [5] are in good agreement



Figure 7 (•) Experimental data [5] and (----) the present prediction of the stress-displacement relationship during compressive loading and unloading for Nicalon-glass.  $\sigma_{\rm d} = -78$  MPa,  $\sigma_{\rm c} = -21.5$  MPa,  $\mu = 0.09$ .

with the prediction in the present study for the bonded interface. Modifications of the present analysis to consider the fibre pull-out test are addressed in a separate paper [17].

Finally, it is noted that crack "pop-in" during initiation of debonding, and the residual stress in the axial direction due to interfacial bonding are not considered in the present study. In the presence of crack pop-in, the stress–displacement relationship in Fig. 6a exhibits a discontinuity when the applied stress reaches  $\sigma_d$  and debonding initiates. In the presence of the residual axial stress, this residual stress should be superimposed on the applied stress as the resultant axial stress.

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